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Electron-hole diagrams versus exciton diagrams: the simplest problem on interacting excitons

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Abstract. We study the interaction of an exciton with a distant metal, which is the simplest problem on interacting excitons: The semiconductor and metal electrons being "different" species, we do not have to worry about the tricky consequences of Pauli exclusion between identical carriers, which appear in any other problem on interacting excitons. We show how the exciton absorption, in the presence of semiconductor-metal interaction, can be derived in a very simple and transparent way from an exciton diagram procedure, provided that we use the appropriate exciton-metal interaction vertex, which contains the scattering from an exciton state to another exciton state under a Coulomb excitation. We also show that the resolution of this problem using standard electron-hole diagrams is dreadfully complicated at the lowest order in the semiconductor-metal interaction already, preventing a full calculation of the exciton-metal coupling from this usual technique.

PACS. 71.35.-y Excitons and related phenomena

In many problems dealing with excitons in semiconductor physics, it would be convenient to work with the exciton entity. However, each time we have to take into account interactions with excitons, we must crack the excitons into electrons and holes, and possibly tight them back into excitons after having performed the interactions, as the true physical interactions are always interactions with free electrons and free holes.

If we want to work with the exciton entity, the difficulty – which is a quite major one – comes from the fact that, while electrons or holes are true fermions, the exciton is not a true boson. The fermionic character of these excitons has in fact extremely tricky consequences. Up to now, all the attempts [1–8] to include it ended by some dressed Coulomb interactions. They all miss purely fermionic contributions, impossible to pick up within the procedures they use.

In these previous attempts, the exact semiconductor Hamiltonian is basically replaced by a phenomenological exciton Hamiltonian which is written in terms of exciton-boson operators. If such a replacement were correct, it would be possible to use bosonlike propagators for the excitons and exciton interaction vertices deduced from the interaction term of the phenomenological boson-exciton Hamiltonian. With these exciton propagators and these exciton interaction vertices, it would be then possible to expand problems on interacting excitons in "exciton diagrams", following a standard procedure.

The trouble is that this effective Hamiltonian is barely incorrect: besides the fact that, of course, it relies on truncatures, which could be justified at low exciton density N/V, by considering the lowest order terms in Na_x^3/V (a_x being the exciton Bohr radius), it also misses purely fermionic contributions [9] of the same order in Na_x^3/V. The better proof that such purely fermionic contributions are indeed missing, comes from the fact that these terms are necessary to restore the hermiticity of the effective exciton Hamiltonian quoted by everyone up to now: Strangely enough, this dramatic failure for an effective Hamiltonian has remained unnoticed up to now.

Despite all these real difficulties, we would like to find a way of working with excitons while dealing with interactions, and get out of it exact results, of course! Excitons are (obviously) real bosons to zero order in Na_x^3/V. We may thus hope that it should be possible to use boson-like propagators for these excitons, at least in certain limits, provided that we have a very safe way to determine the interaction vertices which have to be used in order to recover the results obtained from an exact procedure. If such an exact procedure exists, we can however wonder what is the utility of finding out another way to get the same results! As all exact interactions are written in terms of free electrons and free holes, all secure approaches have to use these free carriers. As shown below on the simplest case, the diagrammatic procedure which uses these free electrons and holes turns out to be extremely heavy, even at lowest order. A simpler method is thus highly desirable for both algebraic purpose and physical understanding.

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It is clear that, if instead of two species, the electron and the hole, we manipulate one specie only, the exciton, the diagram appearance and their calculations have to be greatly simplified.

We would like to stress that the exciton propagator we are going to use here depends on the exciton momentum $\mathbf{Q}$ and relative motion state ν only. It is conceptually quite different from a two-particle Green's function [6,10], (often called "exciton propagator" unproperly), which depends on the "in" and "out" momenta of the pair.

In order to see how such an exciton diagram procedure can emerge from the exact description of excitons in terms of free electrons and holes, and how we can use it in a secure way, we are going to start by the study of a specific example which corresponds to the simplest of all possible problems on interacting excitons, namely an exciton created in a 2D quantum well and interacting with the 2D electrons of a metal located at a distance d from the well. The reason for this problem to be the simplest one on interacting excitons, lies in the fact that the semiconductor electrons and the metal electrons are different so that Pauli exclusion between electrons, which generates all the tricky terms associated to the fermionic character of the exciton, do not play any role for "different" electrons. Consequently, we just have here to take into account the Coulomb interaction between carriers and to derive the scattering matrix element it induces between two exciton states.

This paper deals with the *technical* aspect of this problem: We are going to introduce exciton diagrams written with exciton propagators, exciton-photon interaction and exciton- metal interaction. We will explain how these interactions can be deduced exactly (*i.e.* without any truncature) from the bare interactions with electrons and holes. In order to prove the validity of the procedure, we will compare its results to the ones obtained in a standard way, *i.e.* with electron-hole (e-h) diagrams made of electron propagators, hole propagators, e-h interaction as well as electron-metal and hole-metal interactions. The interesting physics associated to this problem, *i.e.* the changes in the exciton absorption lineshapes induced by the presence of the metal, and their link with Fermi edge singularities are discussed elsewhere [11]. It could have been possible to put the technical proof of this procedure as a (long) appendix. However, these exciton propagators and exciton diagrams could be of great help in other problems on excitons, provided that they are handled with care. This is why we found inappropriate to hide them at the end of a particular problem.

We want this paper to be the basic paper for a series of works we are going to do on interacting excitons. As we have not found in the literature one textbook [12–14] or review paper [6] we could cite, in which *all* the elementary basics on excitons we need, are derived from scratch in a compact form within the same notations, we start by giving here brief derivations of many useful "well known" results on excitons. The first part of this paper may thus appear somewhat tutorial. In particular, it contains the two equations which link the free e-h pair creation operators to the exciton creation operators, "exciton" meaning the true bound states as well as the diffusive states. These two equations are in fact the key equations for all exact treatment of interactions with excitons. We also show how the semiconductor-photon interaction appears as an exciton-photon interaction in a natural way. We finally show how we can extract from the semiconductor-metal interaction, the exciton-metal vertex we need for exciton diagrams.

In a second part, we consider the photon absorption in the absence of semiconductor-metal interaction and use three different approaches to recover the well known result. The first approach is barely the Fermi golden rule. In the second one, we introduce the exciton diagram procedure and we show how the usual expression of the photon absorption immediately follows from this simplest of all possible exciton diagrams. The third approach is based on standard diagrams with free electrons, free holes, and Coulomb interaction between them. It also contains a rapid derivation of two useful quantities for exact calculations on excitons, namely the renormalized electron-hole interaction and the renormalized semiconductor-photon interaction.

In the last part, we calculate the photon absorption to second order in semiconductor-metal interaction. We use the same three approaches. The first one is again based on the Fermi golden rule to which we add a perturbation expansion procedure. In the second approach, we use exciton diagrams, and show that they lead to the same result in an extremely transparent way. In the third approach, we use standard e-h diagrams which contain the Coulomb interactions between the semiconductor electron and hole, and between them and the metal electrons in all possible ways. This diagrammatic approach is in fact the standard way we can think of, if we want to include the semiconductor-metal Coulomb interaction perturbatively. It usually allows to identify and sum up the dominant processes in an easy way. For this reason, it appeared to us necessary and useful to work it out in details. As expected, this third method gives the result of the two previous ones. We will however see that these e-h diagrams are dreadfully complicated, so that there is no hope to be able to use them for higher order terms in semiconductor-metal interaction.

From this last part, we can reasonably conclude that standard electron-hole diagrams are inappropriate when dealing with exciton interactions: Exciton diagrams are clearly much simpler; they in fact appear as the only possible way to identify and sum up higher order processes. We will use them in the paper dealing with an exciton interacting with a distant metal.

1 Basics on excitons

1.1 Free e-h pairs

A free electron in a box of volume $\mathcal{V}$ ($\mathcal{V} = L^3$ in 3D and $\mathcal{V} = L^2$ in 2D) is characterized by a quantum number $\mathbf{k}_{\rm e}$

quantified in $2\pi/L$; its wave function is

$$\langle \mathbf{r}|\mathbf{k}_{\rm e}\rangle = \langle \mathbf{r}|a^+_{\mathbf{k}_{\rm e}}|v\rangle = \frac{{\rm e}^{{\rm i}\mathbf{k}_{\rm e}\cdot\mathbf{r}}}{\sqrt{\mathcal{V}}}\,, \tag{1.1}$$

$|v\rangle$ being the vacuum state, $a^+_{\mathbf{k}_{\rm e}}$ the $\mathbf{k}_{\rm e}$ free electron creation operator, and $\varepsilon^{\rm (e)}_{\mathbf{k}_{\rm e}} = \frac{\hbar^2 k_{\rm e}^2}{2m_{\rm e}}$ the corresponding free electron energy. Similarly a free hole is characterized by $\mathbf{k}_{\rm h}$, $b^+_{\mathbf{k}_{\rm h}}$ and $\varepsilon^{\rm (h)}_{\mathbf{k}_{\rm h}} = \frac{\hbar^2 k_{\rm h}^2}{2m_{\rm h}}$.

If we now consider a free e-h pair made of one electron $\mathbf{k}_{\rm e}$ and one hole $\mathbf{k}_{\rm h}$, the energy of the corresponding state can be written in terms of the center of mass momentum $\mathbf{q}$ and relative momentum $\mathbf{k}$ as

$$\varepsilon^{\rm (e)}_{\mathbf{k}_{\rm e}} + \varepsilon^{\rm (h)}_{\mathbf{k}_{\rm h}} = \frac{\hbar^2 k^2}{2\mu} + \frac{\hbar^2 q^2}{2M} = \varepsilon_{\mathbf{k}} + E_{\mathbf{q}}, \tag{1.2}$$

where the center of mass and relative masses are given by

$$M = m_{\rm e} + m_{\rm h}, \qquad \frac{1}{\mu} = \frac{1}{m_{\rm e}} + \frac{1}{m_{\rm h}}\,\cdot \tag{1.3}$$

The $\mathbf{q}$ and $\mathbf{k}$ momenta are related to $\mathbf{k}_{\rm e}$ and $\mathbf{k}_{\rm h}$ through $\mathbf{q} = \mathbf{k}_{\rm e} + \mathbf{k}_{\rm h}$ and

$$\frac{\mathbf{k}}{\mu} = \mathbf{v} = \mathbf{v}_{\rm e} - \mathbf{v}_{\rm h} = \frac{\mathbf{k}_{\rm e}}{m_{\rm e}} - \frac{\mathbf{k}_{\rm h}}{m_{\rm h}}, \tag{1.4}$$

so that

$$\begin{array}{l} \mathbf{q} = \mathbf{k}_{\rm e} + \mathbf{k}_{\rm h} \\ \mathbf{k} = \alpha_{\rm h}\mathbf{k}_{\rm e} - \alpha_{\rm e}\mathbf{k}_{\rm h} \end{array} \quad \text{or} \quad \begin{array}{l} \mathbf{k}_{\rm e} = \mathbf{k} + \alpha_{\rm e}\mathbf{q} \\ \mathbf{k}_{\rm h} = -\mathbf{k} + \alpha_{\rm h}\mathbf{q} \end{array} \tag{1.5}$$

where we have set

$$\alpha_{\rm e,h} = \frac{m_{\rm e,h}}{m_{\rm e} + m_{\rm h}}\,\cdot \tag{1.6}$$

(Note that $\alpha_{\rm e}+\alpha_{\rm h} = 1$.) Consequently, a free e-h pair state with relative momentum $\mathbf{k}$ and center of mass momentum $\mathbf{q}$ is given by

$$|\mathbf{k},\mathbf{q}\rangle = a^+_{\mathbf{k}+\alpha_{\rm e}\mathbf{q}} b^+_{-\mathbf{k}+\alpha_{\rm h}\mathbf{q}}|v\rangle. \tag{1.7}$$

This free pair $|\mathbf{k},\mathbf{q}\rangle$ state is eigenstate of the free semiconductor Hamiltonian

$$H_{\rm 0sc} = \sum_{\mathbf{k}} \varepsilon^{\rm (e)}_{\mathbf{k}} a^+_{\mathbf{k}} a_{\mathbf{k}} + \sum_{\mathbf{k}} \varepsilon^{\rm (h)}_{\mathbf{k}} b^+_{\mathbf{k}} b_{\mathbf{k}}, \tag{1.8}$$

with the energy $(\varepsilon_{\mathbf{k}} + E_{\mathbf{q}})$, due to equation (1.2).

1.2 "Bound" e-h pairs or excitons

We now look for the mixtures of these free e-h pair states

$$|X_{\nu,\mathbf{q}}\rangle = \sum_{\mathbf{k}} \phi_\nu(\mathbf{k})|\mathbf{k},\mathbf{q}\rangle = B^+_{\nu,\mathbf{q}}|v\rangle, \tag{1.9}$$

which are eigenstates of the semiconductor Hamiltonian

$$H_{\rm sc} = H_{\rm 0sc} + V_{\rm ee} + V_{\rm hh} + V_{\rm eh}, \tag{1.10}$$

where $V_{\rm ee}$ and $V_{\rm hh}$ are the electron-electron and hole-hole Coulomb interactions, while $V_{\rm eh}$ is the electron-hole interaction:

$$V_{\rm eh} = -\sum_{\mathbf{p},\mathbf{p}',\mathbf{q}} V_{\mathbf{q}} a^+_{\mathbf{p}+\mathbf{q}} b^+_{\mathbf{p}'-\mathbf{q}} b_{\mathbf{p}'} a_{\mathbf{p}}. \tag{1.11}$$

From equations (1.8, 1.9) and (1.11), we get

$$\begin{aligned} H_{\rm sc}|X_{\nu,\mathbf{q}}\rangle &= \sum_{\mathbf{k}} \phi_\nu(\mathbf{k}) \left[(\varepsilon_{\mathbf{k}}+E_{\mathbf{q}})|\mathbf{k},\mathbf{q}\rangle - \sum_{\mathbf{q}'} V_{\mathbf{q}'}|\mathbf{k}+\mathbf{q}',\mathbf{q}\rangle \right] \\ &= E_{\mathbf{q}}|X_{\nu,\mathbf{q}}\rangle + \sum_{\mathbf{k}} \left[\varepsilon_{\mathbf{k}}\phi_\nu(\mathbf{k}) - \sum_{\mathbf{q}''} V_{\mathbf{q}''}\phi_\nu(\mathbf{k}+\mathbf{q}'') \right] |\mathbf{k},\mathbf{q}\rangle, \end{aligned} \tag{1.12}$$

so that we do have

$$H_{\rm sc}|X_{\nu,\mathbf{q}}\rangle = (E_{\mathbf{q}} + \varepsilon_\nu)|X_{\nu,\mathbf{q}}\rangle, \tag{1.13}$$

provided that the $\phi_\nu(\mathbf{k})$'s verify

$$(\varepsilon_{\mathbf{k}} - \varepsilon_\nu)\phi_\nu(\mathbf{k}) = \sum_{\mathbf{q}} V_{\mathbf{q}}\phi_\nu(\mathbf{k}+\mathbf{q}), \tag{1.14}$$

$V_{\mathbf{q}}$ being the Coulomb potential Fourier transform [15]. ($V_{\mathbf{q}} = 4\pi e^2/\mathcal{V}\varepsilon q^2$ in 3D, and $V_{\mathbf{q}} = 2\pi e^2/\mathcal{V}\varepsilon q$ in 2D). Equation (1.14) is just the Fourier transform of

$$\left(\frac{\mathbf{p}^2}{2\mu} - \frac{e^2}{\varepsilon r} \right) |x_\nu\rangle = \varepsilon_\nu |x_\nu\rangle\,\cdot \tag{1.15}$$

This shows that the weight of the $|\mathbf{k},\mathbf{q}\rangle$ free pair state in the $|X_{\nu,\mathbf{q}}\rangle$ exciton is simply

$$\phi_\nu(\mathbf{k}) = \langle \mathbf{k}|x_\nu\rangle\,\cdot \tag{1.16}$$

In 3D, the ν index characterizing the e-h relative motion corresponds to (n,l,m) for bound states, and (K,l,m) for diffusive states, while in 2D, it corresponds to (n,m) and (K,m) respectively.

Equation (1.9) leads us to define the exciton creation operator as

$$B^+_{\nu,\mathbf{q}} = \sum_{\mathbf{k}} \phi_\nu(\mathbf{k}) a^+_{\mathbf{k}+\alpha_{\rm e}\mathbf{q}} b^+_{-\mathbf{k}+\alpha_{\rm h}\mathbf{q}}. \tag{1.17}$$

Let us note that it is indeed natural to divide the total e-h pair momentum $\mathbf{q}$ into $\alpha_{\rm e}\mathbf{q}$ for the electron and $\alpha_{\rm h}\mathbf{q}$ for the hole, as the remaining momentum $\mathbf{k}$ has then a simple physical meaning: It is the momentum of the e-h pair relative motion.

Equation (1.17) can be inverted into

$$a^+_{\mathbf{k}_{\rm e}} b^+_{\mathbf{k}_{\rm h}} = \sum_\nu \phi^*_\nu(\alpha_{\rm h}\mathbf{k}_{\rm e} - \alpha_{\rm e}\mathbf{k}_{\rm h}) B^+_{\nu,\mathbf{k}_{\rm e}+\mathbf{k}_{\rm h}}. \tag{1.18}$$

as can be directly checked by inserting (1.18) into (1.17) and by using

$$\delta_{\nu\nu'} = \langle x_{\nu'}|x_\nu\rangle = \sum_{\mathbf{k}} \phi^*_{\nu'}(\mathbf{k})\phi_\nu(\mathbf{k}). \tag{1.19}$$

Equations (1.17) and (1.18) will appear as the key equations for problems dealing with the exact treatment of interactions involving excitons: Equation (1.17) allows to "open" the exciton into electron and hole, such a separation being necessary to take into account the (exact) interactions written in terms of individual electrons or holes. Once these interactions have acted up, we can "close" back the e-h pairs into excitons by using equation (1.18).

1.3 Exciton-photon interaction

The photon absorption is associated with the excitation of one electron from the valence band to the conduction band, *i.e.* with the creation of one e-h pair. Due to momentum conservation, these pairs must have the same center of mass momentum $\mathbf{Q}$ as the one of the absorbed photon. This leads to write the electron-photon interaction as

$$H_{\mathrm{e-ph}} = \sum_{\mathbf{k}} A a^{+}_{\mathbf{k}+\alpha_{\mathrm{e}}\mathbf{Q}} b^{+}_{-\mathbf{k}+\alpha_{\mathrm{h}}\mathbf{Q}} \alpha_{\mathbf{Q}} + \mathrm{h.c.}, \tag{1.20}$$

$\alpha^{+}_{\mathbf{Q}}$ being the creation operator for a momentum $\mathbf{Q}$ photon. The matrix element A can be approximated by a constant if the transition is allowed by symmetry. Using equation (1.18), we can rewrite this electron-photon interaction as

$$H_{\mathrm{e-ph}} = A \sum_{\mathbf{k},\nu} \phi^{*}_{\nu}(\mathbf{k}) B^{+}_{\nu,\mathbf{Q}} \alpha_{\mathbf{Q}} + \mathrm{h.c.} \tag{1.21}$$

This shows that the electron-photon interaction can also be seen as an exciton-photon interaction:

$$H_{\mathrm{e-ph}} \equiv H_{X-\mathrm{ph}} = \sum_{\nu} A_{\nu} B^{+}_{\nu,\mathbf{Q}} \alpha_{\mathbf{Q}} + \mathrm{h.c.} \tag{1.22}$$

with a coupling constant A_ν which depends on ν through

$$\begin{aligned} A_{\nu} &= A \sum_{\mathbf{k}'} \phi^{*}_{\nu}(\mathbf{k}') \\ &= A \sum_{\mathbf{k}'} \langle x_{\nu}|\mathbf{k}'\rangle \langle \mathbf{k}'|\mathbf{r}=\mathbf{0}\rangle \sqrt{\mathcal{V}} = A\sqrt{\mathcal{V}} \langle x_{\nu}|\mathbf{r}=\mathbf{0}\rangle \cdot \end{aligned} \tag{1.23}$$

The absorption processes corresponding to equation (1.22) are shown in Figure 1a.

Note that we could as well write equation (1.20) in terms of $a^{+}_{\mathbf{k}+\mathbf{Q}} b^{+}_{-\mathbf{k}}$. Of course we would get out of it the same exciton-photon interaction, equations (1.22, 1.23). However, here again, it is more physical to "divide" the photon momentum $\mathbf{Q}$ into $\alpha_{\mathrm{e}}\mathbf{Q}$ for the electron and $\alpha_{\mathrm{h}}\mathbf{Q}$ for the hole, as the remaining momentum $\mathbf{k}$ has then a physical meaning.

1.4 Semiconductor-metal interaction

We now consider a semiconductor located in a 2D quantum well at a distance d from a doped quantum well acting as a 2D metal. The metal Hamiltonian reads

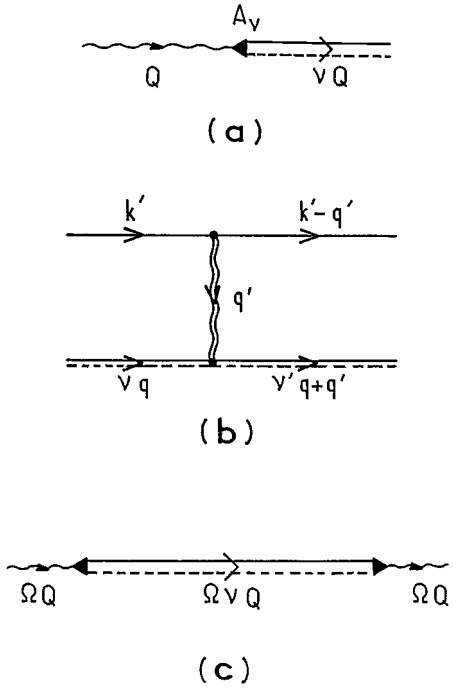

Fig. 1. (a) Exciton-photon interaction as given by equation (1.22). (b) Exciton-metal interaction as given by equation (1.35). (c) Response function in the absence of exciton-metal interaction, as given by exciton diagrams (Eq. (2.8)).

$$\begin{aligned} H_{\mathrm{m}} &= H_{0\mathrm{m}} + V_{\mathrm{mm}} \\ H_{0\mathrm{m}} &= \sum_{\mathbf{k}} \varepsilon^{(\mathrm{m})}_{\mathbf{k}} c^{+}_{\mathbf{k}} c_{\mathbf{k}} \\ V_{\mathrm{mm}} &= \frac{1}{2} \sum_{\mathbf{q},\mathbf{k},\mathbf{k}'} V_{\mathbf{q}} c^{+}_{\mathbf{k}+\mathbf{q}} c^{+}_{\mathbf{k}'-\mathbf{q}} c_{\mathbf{k}'} c_{\mathbf{k}}, \end{aligned} \tag{1.24}$$

where $c^{+}_{\mathbf{k}}$ creates a $\mathbf{k}$ electron in the metal. Note that, as metal electrons and semiconductor electrons are different, their creation operators have to be labelled by different letters.

The metal electrons interact with each other through V_{mm}. They also interact with the electrons and the holes of the semiconductor through electron-metal and hole-metal interactions. For 2D semiconductor electrons located at $\mathbf{r}_n$ and 2D metal electrons located at $\tilde{\mathbf{r}}_m$, the two planes being at a distance d, the electron-metal interaction [15] is given by

$$\sum_{n,m} \frac{e^2}{\varepsilon \sqrt{(\mathbf{r}_n - \tilde{\mathbf{r}}_m)^2 + d^2}} \cdot \tag{1.25}$$

In second quantization, this interaction reads

$$V_{\mathrm{em}} = \sum_{\mathbf{q},\mathbf{k},\mathbf{k}'} \tilde{V}_{\mathbf{q}} a^{+}_{\mathbf{k}+\mathbf{q}} c^{+}_{\mathbf{k}'-\mathbf{q}} c_{\mathbf{k}'} a_{\mathbf{k}}. \tag{1.26}$$

From the standard procedure to rewrite an operator in second quantization, we get the coupling constant $\tilde{V}_{\mathbf{q}}$ as

$$\tilde{V}_{\mathbf{q}} = \iint_S \frac{\mathrm{d}^2 r}{S} \mathrm{e}^{\mathrm{i}\mathbf{q}\cdot\mathbf{r}} \frac{e^2}{\varepsilon\sqrt{r^2+d^2}} = \frac{2\pi e^2}{S\varepsilon q}\mathrm{e}^{-qd} = V_{\mathbf{q}}\mathrm{e}^{-qd}. \tag{1.27}$$

Similarly the hole-metal interaction is given by

$$V_{\mathrm{hm}} = -\sum_{\mathbf{q},\mathbf{k},\mathbf{k}'} \tilde{V}_{\mathbf{q}} b^+_{\mathbf{k}+\mathbf{q}} c^+_{\mathbf{k}'-\mathbf{q}} c_{\mathbf{k}'} b_{\mathbf{k}}. \tag{1.28}$$

1.5 Exciton-metal coupling

There is clearly no exact way to write $V_{\mathrm{em}}+V_{\mathrm{hm}}$ in terms of $B^+_{\nu,\mathbf{q}}$. Nevertheless, it is possible to find out the exact exciton change induced by $V_{\mathrm{em}}+V_{\mathrm{hm}}$ as well as the corresponding change of the metal Fermi sea.

Let us consider the action of V_{em} on a state composed of the exciton $|X_{\nu,\mathbf{q}}\rangle$ and the metal in the excited Fermi sea state $|M\rangle$. From equation (1.26) we get

$$V_{\mathrm{em}}|X_{\nu,\mathbf{q}}\rangle\otimes|M\rangle = \sum_{\mathbf{q}'}\tilde{V}_{\mathbf{q}'}\sum_{\mathbf{k}} a^+_{\mathbf{k}+\mathbf{q}'}a_{\mathbf{k}}B^+_{\nu,\mathbf{q}}|v\rangle\otimes\sum_{\mathbf{k}'}c^+_{\mathbf{k}'-\mathbf{q}'}c_{\mathbf{k}'}|M\rangle. \tag{1.29}$$

By "opening" the exciton into e-h pairs through equation (1.17), we find

$$\begin{aligned}\sum_{\mathbf{k}} a^+_{\mathbf{k}+\mathbf{q}'}a_{\mathbf{k}}B^+_{\nu,\mathbf{q}}|v\rangle &= \sum_{\mathbf{k},\mathbf{p}}\phi_\nu(\mathbf{p})a^+_{\mathbf{k}+\mathbf{q}'}a_{\mathbf{k}}a^+_{\mathbf{p}+\alpha_{\mathrm{e}}\mathbf{q}}b^+_{-\mathbf{p}+\alpha_{\mathrm{h}}\mathbf{q}}|v\rangle \\ &= \sum_{\mathbf{k}}\phi_\nu(\mathbf{k}-\alpha_{\mathrm{e}}\mathbf{q})a^+_{\mathbf{k}+\mathbf{q}'}b^+_{-\mathbf{k}+\mathbf{q}}|v\rangle.\end{aligned} \tag{1.30}$$

We then "close" back these e-h pairs into excitons through equation (1.18). This gives

$$\sum_{\mathbf{k}}\phi_\nu(\mathbf{k}-\alpha_{\mathrm{e}}\mathbf{q})a^+_{\mathbf{k}+\mathbf{q}'}b^+_{-\mathbf{k}+\mathbf{q}}|v\rangle = \sum_{\mathbf{k},\nu'}\phi_\nu(\mathbf{k}-\alpha_{\mathrm{e}}\mathbf{q})\phi^*_{\nu'}(\mathbf{k}-\alpha_{\mathrm{e}}\mathbf{q}+\alpha_{\mathrm{h}}\mathbf{q}')B^+_{\nu',\mathbf{q}+\mathbf{q}'}|v\rangle, \tag{1.31}$$

so that we finally get

$$V_{\mathrm{em}}|X_{\nu,\mathbf{q}}\rangle\otimes|M\rangle = \sum_{\mathbf{q}',\nu'}\tilde{V}_{\mathbf{q}'}\gamma^{(\mathrm{e})}_{\nu'\nu}(\mathbf{q}')|X_{\nu',\mathbf{q}+\mathbf{q}'}\rangle\otimes\sum_{\mathbf{k}'}c^+_{\mathbf{k}'-\mathbf{q}'}c_{\mathbf{k}'}|M\rangle \tag{1.32}$$

where the coupling constant $\gamma^{(\mathrm{e})}_{\nu'\nu}(\mathbf{q}')$ is given by

$$\gamma^{(\mathrm{e})}_{\nu'\nu}(\mathbf{q}') = \sum_{\mathbf{k}'}\phi^*_{\nu'}(\mathbf{k}'+\alpha_{\mathrm{h}}\mathbf{q}')\phi_\nu(\mathbf{k}') = \langle x_{\nu'}|\mathrm{e}^{\mathrm{i}\alpha_{\mathrm{h}}\mathbf{q}'\cdot\mathbf{r}}|x_\nu\rangle \tag{1.33}$$

as can be checked by inserting closure relations for $|\mathbf{p}\rangle$ states on both sides of the exponential.

A similar equation is obtained for the action of V_{hm} on $|X_{\nu,\mathbf{q}}\rangle\otimes|M\rangle$, with $\gamma^{(\mathrm{e})}_{\nu'\nu}(\mathbf{q}')$ replaced by $-\gamma^{(\mathrm{h})}_{\nu'\nu}(\mathbf{q}')$, given by

$$\gamma^{(\mathrm{h})}_{\nu'\nu}(\mathbf{q}') = \langle x_{\nu'}|\mathrm{e}^{-\mathrm{i}\alpha_{\mathrm{e}}\mathbf{q}'\cdot\mathbf{r}}|x_\nu\rangle\cdot \tag{1.34}$$

If we now consider the action of $V_{\mathrm{em}}+V_{\mathrm{hm}}$ on the same state, we find

$$(V_{\mathrm{em}}+V_{\mathrm{hm}})|X_{\nu,\mathbf{q}}\rangle\otimes|M\rangle = \sum_{\mathbf{q}',\nu'}\tilde{V}_{\mathbf{q}'}\gamma_{\nu'\nu}(\mathbf{q}')|X_{\nu',\mathbf{q}+\mathbf{q}'}\rangle\otimes\sum_{\mathbf{k}'}c^+_{\mathbf{k}'-\mathbf{q}'}c_{\mathbf{k}'}|M\rangle. \tag{1.35}$$

The "Coulomb interaction" between the exciton and the metal appears as the product of the bare Coulomb interaction $\tilde{V}_{\mathbf{q}'}$ by

$$\gamma_{\nu'\nu}(\mathbf{q}') = \langle x_{\nu'}|\mathrm{e}^{\mathrm{i}\alpha_{\mathrm{h}}\mathbf{q}'\cdot\mathbf{r}} - \mathrm{e}^{-\mathrm{i}\alpha_{\mathrm{e}}\mathbf{q}'\cdot\mathbf{r}}|x_\nu\rangle, \tag{1.36}$$

which describes the scattering of the exciton from a ν to a ν' state under a $\mathbf{q}'$ excitation.

Equation (1.35) shows that $V_{\mathrm{em}}+V_{\mathrm{hm}}$ transforms the exciton $|X_{\nu,\mathbf{q}}\rangle$ into a set of other excitons, the change of the center of mass momentum $\mathbf{q}'$ being compensated by the momentum change $(-\mathbf{q}')$ in the metal Fermi sea, as expected. The processes corresponding to equation (1.35) are represented in Figure 1b.

2 Absorption in the absence of exciton-metal coupling

Let us start by the calculation of the semiconductor absorption when the metal is far away, so that it does not interact with the semiconductor. We are going to do it in three different ways, as these three approaches turn out to be quite complementary for the understanding of what happens when the excitons interact with other carriers.

2.1 Approach without diagrams

The Fermi golden rule gives the absorption rate for a photon of momentum $\mathbf{Q}$ and energy Ω, as

$$\mathcal{A}^{(0)}(\Omega,\mathbf{Q}) = \frac{2\pi}{\hbar}\sum_f |\langle f|H_{\mathrm{e-ph}}|i\rangle|^2\delta(E_f-E_i-\Omega), \tag{2.1}$$

where the initial and final states $|i\rangle$ and $|f\rangle$ are eigenstates of the semiconductor Hamiltonian H_{sc}. We can make the $|f\rangle$ states formally disappearing from equation (2.1) by rewriting $\mathcal{A}^{(0)}(\Omega,\mathbf{Q})$ as

$$\mathcal{A}^{(0)}(\Omega,\mathbf{Q}) = -\frac{2}{\hbar}\mathrm{Im}S^{(0)}(\Omega,\mathbf{Q}) \tag{2.2}$$

$$S^{(0)}(\Omega,\mathbf{Q}) = \langle i|H_{\mathrm{e-ph}}\frac{1}{\Omega+E_i-H_{\mathrm{sc}}+\mathrm{i}\eta}H_{\mathrm{e-ph}}|i\rangle\cdot \tag{2.3}$$

(One trivially goes from Eq. (2.3) to Eq. (2.1) by inserting two closure relations for $H_{\rm sc}$ eigenstates on each side of the fraction.)

If the initial state $|i\rangle$ is the semiconductor ground state, *i.e.* the vacuum state for e-h pairs, we have, due to equation (1.22):

$$H_{\rm e-ph}|i\rangle = H_{X-{\rm ph}}|v\rangle = \sum_\nu A_\nu |X_{\nu,\mathbf{Q}}\rangle, \tag{2.4}$$

(where we have dropped the photon part which is in fact included in the Ω appearing in Eq. (2.1)). From equation (2.3), we immediately find

$$S^{(0)}(\Omega,\mathbf{Q}) = \sum_\nu \frac{|A_\nu|^2}{\Omega - \varepsilon_\nu - E_{\mathbf{Q}} + {\rm i}\eta}, \tag{2.5}$$

in which we have chosen the zero energy to be the ground state energy E_i. By taking the imaginary part of $S^{(0)}(\Omega,\mathbf{Q})$, we do recover the well known semiconductor absorption, composed of exciton lines at $\Omega = \varepsilon_\nu + E_Q \simeq \varepsilon_\nu$ (since $\mathbf{Q}$ is very small), the corresponding line weight being

$$|A_\nu|^2 = A^2\mathcal{V}|\langle \mathbf{r}=\mathbf{0}|x_\nu\rangle|^2. \tag{2.6}$$

2.2 Exciton diagrams

The exciton-photon interaction, given in equation (1.22), is represented by Figure 1a. It may appear as reasonable to add an Ω variable, usual for diagrams, and to impose an $(\Omega,\mathbf{Q})$ conservation at each interaction vertex. If we introduce an exciton propagator given by

$$G_\nu(\Omega,\mathbf{Q}) = \frac{1}{\Omega - (\varepsilon_\nu + E_{\mathbf{Q}}) + {\rm i}\eta}, \tag{2.7}$$

which is quite reasonable for an object of energy $(\varepsilon_\nu + E_{\mathbf{Q}})$, the response function $S^{(0)}(\Omega,\mathbf{Q})$, represented in Figure 1c, reads

$$S^{(0)}(\Omega,\mathbf{Q}) = \sum_\nu |A_\nu|^2 G_\nu(\Omega,\mathbf{Q}), \tag{2.8}$$

which is nothing but equation (2.5).

2.3 Electron-hole diagrams

2.3.1 Electron and hole propagators

In the case of a semiconductor having no e-h pairs, the electron and hole propagators are given by

$$g_{\rm e,h}(\omega,\mathbf{k}) = \frac{1}{\omega - \varepsilon^{\rm (e,h)}_{\mathbf{k}} + {\rm i}\eta}, \tag{2.9}$$

(iη has to be replaced [13,14] by iη sign$(\varepsilon_k - \mu)$ if the semiconductor already has e-h pairs up to a chemical potential μ). In most problems on excitons, these electron and hole propagators appear through the combination [6,12]

$$\begin{aligned} G_{\rm eh}(\omega,\mathbf{k},\mathbf{q}) &= \int \frac{{\rm i}d\omega_1}{2\pi} g_{\rm e}(\omega_1+\omega, \mathbf{k}+\alpha_{\rm e}\mathbf{q}) g_{\rm h}(-\omega_1, -\mathbf{k}+\alpha_{\rm h}\mathbf{q}) \\ &= \frac{1}{\omega - \varepsilon^{\rm (e)}_{\mathbf{k}+\alpha_{\rm e}\mathbf{q}} - \varepsilon^{\rm (h)}_{-\mathbf{k}+\alpha_{\rm h}\mathbf{q}} + {\rm i}\eta} \\ &= \frac{1}{\omega - \varepsilon_{\mathbf{k}} - E_{\mathbf{q}} + {\rm i}\eta}. \end{aligned} \tag{2.10}$$

$G_{\rm eh}(\omega,\mathbf{k},\mathbf{q})$ can be seen as the propagator of a free e-h pair of energy $(\varepsilon_{\mathbf{k}} + E_{\mathbf{q}})$.

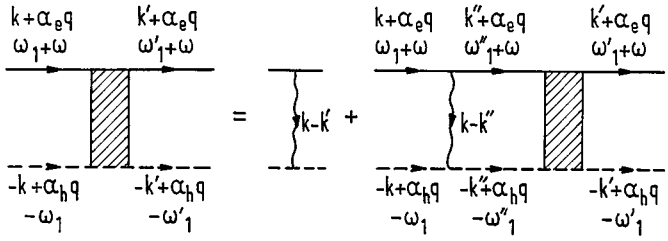

Fig. 2. Dyson equation satisfied by the renormalized e-h interaction, as given by equation (2.11).

2.3.2 Renormalized e-h Coulomb interaction and electron-photon interaction

The only thing one electron and one hole can do is to repeatedly interact *via* Coulomb interaction. From these repeated Coulomb interactions, we can construct a renormalized e-h Coulomb interaction and a renormalized electron-photon interaction.

(i) The first one, shown in Figure 2, verifies the Dyson equation

$$\begin{aligned} W_{\mathbf{k},\mathbf{k}'}(\omega,q) = &-V_{\mathbf{k}-\mathbf{k}'} \\ &- \sum_{\mathbf{k}''} V_{\mathbf{k}-\mathbf{k}''} G_{\rm eh}(\omega,\mathbf{k}'',\mathbf{q}) W_{\mathbf{k}'',\mathbf{k}'}(\omega,\mathbf{q}). \end{aligned} \tag{2.11}$$

We can solve this integral equation by expanding $V_{\mathbf{k}-\mathbf{k}'}$, taken as a function of $\mathbf{k}$, on the $\phi_\nu(\mathbf{k})$ functions which can serve as a basis for $\mathbf{k}$ functions. This leads to

$$V_{\mathbf{k}-\mathbf{k}'} = \sum_\nu v_\nu \phi^*_\nu(\mathbf{k}). \tag{2.12}$$

The same $\phi_\nu(\mathbf{k})$ basis can be used to expand $L(\mathbf{k})$ defined as

$$L(\mathbf{k}) = G_{\rm eh}(\omega,\mathbf{k},\mathbf{q}) W_{\mathbf{k},\mathbf{k}'}(\omega,\mathbf{q}) = \sum_\nu l_\nu \phi^*_\nu(\mathbf{k}). \tag{2.13}$$

We insert these expansions into the Dyson equation (2.11), and we use equation (2.10) for $G_{\rm eh}$. This leads to

$$\begin{aligned} (\omega - \varepsilon_{\mathbf{k}} - E_{\mathbf{q}} + {\rm i}\eta) \sum_\nu l_\nu \phi^*_\nu(\mathbf{k}) = \\ - \sum_\nu v_\nu \phi^*_\nu(\mathbf{k}) - \sum_{\mathbf{k}'',\nu} V_{\mathbf{k}-\mathbf{k}''} l_\nu \phi^*_\nu(\mathbf{k}''). \end{aligned} \tag{2.14}$$

The sum over $\mathbf{k}''$ can be obtained from equation (1.14), so that the above equation gives

$$\sum_{\nu}[l_{\nu}(\omega-\varepsilon_{\nu}-E_{\mathbf{q}}+\mathrm{i}\eta)+v_{\nu}]\phi_{\nu}^{*}(\mathbf{k})=0, \qquad (2.15)$$

which should be verified for all $\mathbf{k}$. This imposes l_{ν} to be given by

$$l_{\nu}=-\frac{v_{\nu}}{\omega-\varepsilon_{\nu}-E_{\mathbf{q}}+\mathrm{i}\eta}=-v_{\nu}G_{\nu}(\omega,\mathbf{q}). \qquad (2.16)$$

We then note that equation (2.12) can be inverted as

$$\begin{aligned} v_{\nu}&=\sum_{\mathbf{k}}V_{\mathbf{k}-\mathbf{k}'}\phi_{\nu}(\mathbf{k})=(\varepsilon_{\mathbf{k}'}-\varepsilon_{\nu})\phi_{\nu}(\mathbf{k}')\\ &=\left[G_{\nu}^{-1}(\omega,\mathbf{q})-G_{\mathrm{eh}}^{-1}(\omega,\mathbf{k}',\mathbf{q})\right]\phi_{\nu}(\mathbf{k}'), \end{aligned} \qquad (2.17)$$

due to equation (1.14), so that, from equations (2.13, 2.16, 2.17), we finally get the renormalized e-h Coulomb interaction [6] as

$$\begin{aligned} W_{\mathbf{k},\mathbf{k}'}(\omega,\mathbf{q})=&\frac{1}{G_{\mathrm{eh}}(\omega,\mathbf{k},\mathbf{q})}\sum_{\nu}G_{\nu}(\omega,\mathbf{q})\\ &\times\left[\frac{1}{G_{\mathrm{eh}}(\omega,\mathbf{k}',\mathbf{q})}-\frac{1}{G_{\nu}(\omega,\mathbf{q})}\right]\phi_{\nu}^{*}(\mathbf{k})\phi_{\nu}(\mathbf{k}')\\ =&\frac{1}{G_{\mathrm{eh}}(\omega,\mathbf{k},\mathbf{q})}\left[-\delta_{\mathbf{k}\mathbf{k}'}+\frac{1}{G_{\mathrm{eh}}(\omega,\mathbf{k}',\mathbf{q})}\right.\\ &\left.\times\sum_{\nu}G_{\nu}(\omega,\mathbf{q})\phi_{\nu}^{*}(\mathbf{k})\phi_{\nu}(\mathbf{k}')\right]. \end{aligned} \qquad (2.18)$$

(ii) We now turn to the renormalized electron-photon interaction [12] shown in Figure 3. The "in" interaction reads

$$A_{\mathbf{k}}^{(\mathrm{in})}(\Omega,\mathbf{Q})=A+A\sum_{\mathbf{k}'}G_{\mathrm{eh}}(\Omega,\mathbf{k}',\mathbf{Q})W_{\mathbf{k}',\mathbf{k}}(\Omega,\mathbf{Q}). \qquad (2.19)$$

By using $W_{\mathbf{k}',\mathbf{k}}$ as given by equation (2.18) and the definition of A_{ν} given in equation (1.23), we get

$$A_{\mathbf{k}}^{(\mathrm{in})}(\Omega,\mathbf{Q})=\frac{1}{G_{\mathrm{eh}}(\Omega,\mathbf{k},\mathbf{Q})}\sum_{\nu}A_{\nu}\phi_{\nu}(\mathbf{k})G_{\nu}(\Omega,\mathbf{Q}). \qquad (2.20)$$

A similar calculation gives the "out" interaction as

$$\begin{aligned} A_{\mathbf{k}}^{(\mathrm{out})}(\Omega,\mathbf{Q})&=A^{*}+A^{*}\sum_{\mathbf{k}'}G_{\mathrm{eh}}(\Omega,\mathbf{k}',\mathbf{Q})W_{\mathbf{k},\mathbf{k}'}(\Omega,\mathbf{Q})\\ &=\frac{1}{G_{\mathrm{eh}}(\Omega,\mathbf{k},\mathbf{Q})}\sum_{\nu}A_{\nu}^{*}\phi_{\nu}^{*}(\mathbf{k})G_{\nu}(\Omega,\mathbf{Q}). \end{aligned} \qquad (2.21)$$

Note that $A_{\mathbf{k}}^{(\mathrm{in})}$ and $A_{\mathbf{k}}^{(\mathrm{out})}$ are not complex conjugate since G_{eh} and G_{ν} are not real.

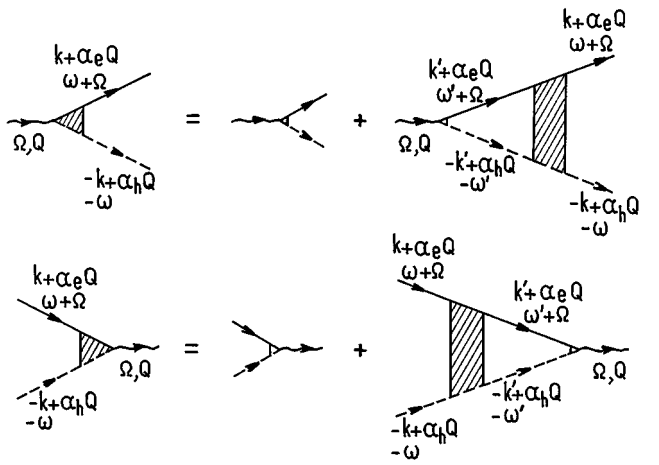

Fig. 3. Dyson equation satisfied by the renormalized semiconductor-photon interaction for "in" and "out" processes, as given by equations (2.19) and (2.21) respectively.

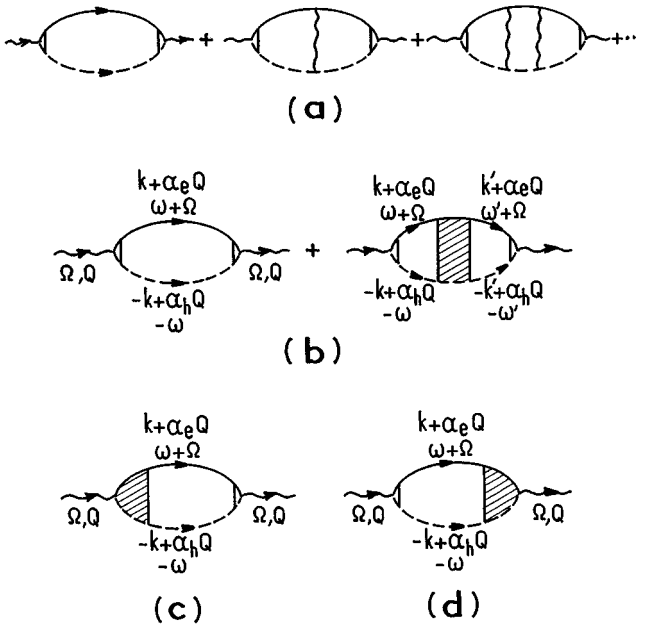

Fig. 4. (a) Set of diagrams leading to the response function in the absence of exciton-metal interaction. (b) Summation of these diagrams by using the renormalized e-h interaction (Eq. (2.22)). (c) (resp. (d)) Summation of these diagrams by using the renormalized "in" (resp. "out") semiconductor-photon interaction (Eq. (2.23)).

2.3.3 Absorption diagrams

The diagrammatic expansion of the response function corresponds to the set of ladder diagrams shown in Figure 4a. They can be summed up by using either $W_{\mathbf{k},\mathbf{k}'}$ or $A_{\mathbf{k}}^{(\mathrm{in})}$ or $A_{\mathbf{k}}^{(\mathrm{out})}$.

- These ladder diagrams correspond to the two diagrams of Figure 4b. They are thus given by

$$S^{(0)}(\Omega,\mathbf{Q}) = AA^*\Bigg[\sum_{\mathbf{k}} G_{\rm eh}(\Omega,\mathbf{k},\mathbf{Q}) + \sum_{\mathbf{k},\mathbf{k}'} G_{\rm eh}(\Omega,\mathbf{k},\mathbf{Q}) \times W_{\mathbf{k},\mathbf{k}'}(\Omega,\mathbf{Q}) G_{\rm eh}(\Omega,\mathbf{k}',\mathbf{Q})\Bigg]$$
$$= AA^*\sum_{\mathbf{k},\mathbf{k}'} G_{\rm eh}(\Omega,\mathbf{k},\mathbf{Q}) \times \left[\delta_{\mathbf{k},\mathbf{k}'} + W_{\mathbf{k},\mathbf{k}'}(\Omega,\mathbf{Q}) G_{\rm eh}(\Omega,\mathbf{k}',\mathbf{Q})\right]. \quad (2.22)$$

  From equation (2.18), we see that the $\delta_{\mathbf{k}\mathbf{k}'}$ terms disappear from the bracket and the remaining term gives the same result as the one previously obtained in equation (2.8).
- The same set of ladder diagrams also corresponds to one of the two diagrams of Figures 4c and d. Using equations (2.20) or (2.21) we get [12]

$$S^{(0)}(\Omega,\mathbf{Q}) = A^*\sum_{\mathbf{k}} A^{(\rm in)}_{\mathbf{k}}(\Omega,\mathbf{Q}) G_{\rm eh}(\Omega,\mathbf{k},\mathbf{Q})$$
$$= A\sum_{\mathbf{k}} A^{(\rm out)}_{\mathbf{k}}(\Omega,\mathbf{Q}) G_{\rm eh}(\Omega,\mathbf{k},\mathbf{Q})$$
$$= \sum_{\nu} |A_\nu|^2 G_\nu(\Omega,\mathbf{Q}), \quad (2.23)$$

i.e. the same result again.

We see that, while the three approaches hopefully give the same result, the exciton diagram procedure already appears as being the simplest one, both visually and algebraically. The difference is even more dramatic when the problem contains more carriers than the photocreated e-h pair, as we now show.

3 Photon absorption to lowest order in semiconductor-metal interaction

We now want to include the electron-metal and hole-metal interaction to lowest order (which will turn out to be second order). We will however neglect Coulomb interaction between the metal electrons for simplicity: It just changes the internal readjustment of the metal to an external excitation, but does not affect the technical difficulty of the problem resulting from the additional interaction felt by the electron and hole making the exciton.

Here again we derive the result by the same three procedures we previously used.

3.1 Approach without diagrams

We must now consider a system made of the semiconductor and the metal, in interaction with each other. The total Hamiltonian is thus

$$H = H_0 + V_{\rm em} + V_{\rm hm} \qquad \text{with} \qquad H_0 = H_{\rm sc} + H_{0\rm m}, \quad (3.1)$$

if we forget $V_{\rm mm}$. The response function for such a system is given by

$$S(\Omega,\mathbf{Q}) = \langle i|H_{\rm e-ph}\frac{1}{\Omega - H + {\rm i}\eta}H_{\rm e-ph}|i\rangle, \quad (3.2)$$

where the initial state is now

$$|i\rangle = |v\rangle\otimes|M_0\rangle, \quad (3.3)$$

$|M_0\rangle$ being the metal ground state. We will take again the zero energy such that $E_i = 0$. The response function $S(\Omega,\mathbf{Q})$ can be expanded in $V = (V_{\rm em} + V_{\rm hm})$ by using the well known expansion procedure:

$$\frac{1}{a-H} = \frac{1}{a-H_0} + \frac{1}{a-H_0}V\frac{1}{a-H_0} + \frac{1}{a-H_0}V\frac{1}{a-H_0}V\frac{1}{a-H_0} + \cdots \quad (3.4)$$

- The zero order term simply gives $S^{(0)}(\Omega,\mathbf{Q})$ as calculated in the absence of metal electrons.
- The first order term gives zero: As $V_{\rm em} + V_{\rm hm}$ creates one e-h pair in the metal Fermi sea, we do have $\langle M_0|V_{\rm em} + V_{\rm hm}|M_0\rangle = 0$.
- The second order term, which is thus the lowest order one, can be written as

$$S^{(2)}(\Omega,\mathbf{Q}) = \left\langle \Psi^{(1)}|\frac{1}{\Omega - H_0 + {\rm i}\eta}|\Psi^{(1)}\right\rangle, \quad (3.5)$$

$$|\Psi^{(1)}\rangle = (V_{\rm em} + V_{\rm hm})|\Psi^{(0)}\rangle,$$
$$|\Psi^{(0)}\rangle = \frac{1}{\Omega - H_0 + {\rm i}\eta}H_{\rm e-ph}|v\rangle\otimes|M_0\rangle\cdot \quad (3.6)$$

From equations (2.4) and (2.7), we immediately get

$$|\Psi^{(0)}\rangle = \sum_{\nu} A_\nu G_\nu(\Omega,\mathbf{Q})|X_{\nu,\mathbf{Q}}\rangle\otimes|M_0\rangle\cdot \quad (3.7)$$

Then, we use equation (1.35) to get the action of $(V_{\rm em} + V_{\rm hm})$ on the above ket. This gives $|\Psi^{(1)}\rangle$ as

$$|\Psi^{(1)}\rangle \sum_{\nu,\nu_1}\sum_{\mathbf{q},\mathbf{p}} A_\nu G_\nu(\Omega,\mathbf{Q}) \times \tilde{V}_{\mathbf{q}}\gamma_{\nu_1\nu}(\mathbf{q})|X_{\nu_1,\mathbf{Q}+\mathbf{q}}\rangle\otimes c^+_{\mathbf{p}-\mathbf{q}}c_{\mathbf{p}}|M_0\rangle\cdot \quad (3.8)$$

The metal part differs from zero for $\mathbf{p}$ inside and $\mathbf{p}-\mathbf{q}$ outside the Fermi sea only.

In $S^{(2)}(\Omega,\mathbf{Q})$ thus appears

$$\langle M_0|c^+_{\mathbf{p}'}c_{\mathbf{p}'-\mathbf{q}'}\otimes\langle X_{\nu_2,\mathbf{Q}+\mathbf{q}'}|\frac{1}{\Omega-H_0+\mathrm{i}\eta}|X_{\nu_1,\mathbf{Q}+\mathbf{q}}\rangle \otimes c^+_{\mathbf{p}-\mathbf{q}}c_{\mathbf{p}}|M_0\rangle = \frac{\delta_{\mathbf{p}\mathbf{p}'}\delta_{\mathbf{q}\mathbf{q}'}\delta_{\nu_1\nu_2}}{\Omega-(\varepsilon_{\nu_1}+E_{\mathbf{Q}+\mathbf{q}}+\varepsilon^{(\mathrm{m})}_{\mathbf{p}-\mathbf{q}}-\varepsilon^{(\mathrm{m})}_{\mathbf{p}})+\mathrm{i}\eta}\cdot \tag{3.9}$$

So that the second order response function is finally given by

$$S^{(2)}(\Omega,\mathbf{Q}) = \sum_{\nu,\nu',\nu_1}\sum_{\mathbf{q}} A^*_\nu A_{\nu'}\tilde{V}^2_{\mathbf{q}}\gamma_{\nu\nu_1}(\mathbf{q})\gamma_{\nu_1\nu'}(-\mathbf{q})G_\nu(\Omega,\mathbf{Q})G_{\nu'}(\Omega,\mathbf{Q}) \times \sum_{\substack{\mathbf{p}\,\mathrm{in}\\ \mathbf{p}-\mathbf{q}\,\mathrm{out}}} G_{\nu_1}(\Omega+\varepsilon^{(\mathrm{m})}_{\mathbf{p}}-\varepsilon^{(\mathrm{m})}_{\mathbf{p}-\mathbf{q}},\mathbf{Q}+\mathbf{q}), \tag{3.10}$$

since $\gamma^*_{\nu'\nu_1}(\mathbf{q})=\gamma_{\nu_1\nu'}(-\mathbf{q})$ (see Eq. (1.36)). We note that while the bare interaction $(V_{\mathrm{em}}+V_{\mathrm{hm}})$ acts on the individual electron or hole inside the exciton, the final result indeed reads in terms of exciton energies only.

3.2 Exciton diagrams

We now recover the same result by using exciton diagrams. The second order diagram in the exciton-metal interaction is shown in Figure 5. In it, we have used the $(\omega,\mathbf{q})$ conservation at each interaction vertex. This diagram is thus given by

$$S^{(2)}(\Omega,\mathbf{Q}) = \sum_{\nu,\nu',\nu_1}\sum_{\mathbf{q}} A_\nu A^*_{\nu'}\tilde{V}^2_{\mathbf{q}}\gamma_{\nu'\nu_1}(-\mathbf{q}) \times\gamma_{\nu_1\nu}(\mathbf{q})G_\nu(\Omega,\mathbf{Q})G_{\nu'}(\Omega,\mathbf{Q}) \times\int\frac{\mathrm{id}\omega}{2\pi}G_{\nu_1}(\Omega+\omega,\mathbf{Q}+\mathbf{q})B_0(\omega,\mathbf{q}), \tag{3.11}$$

where $B_0(\omega,\mathbf{q})$ is the usual "bubble" contribution. In terms of the metal electron propagator

$$g_{\mathrm{m}}(\omega,\mathbf{p}) = \frac{1}{\omega-\varepsilon^{(\mathrm{m})}_{\mathbf{p}}+\mathrm{i}\eta\mathrm{sign}\left(\varepsilon^{(\mathrm{m})}_{\mathbf{p}}-\mu\right)}, \tag{3.12}$$

this "bubble" contribution is given by

$$B_0(\omega,\mathbf{p}) = -\sum_{\mathbf{p}}\int\frac{\mathrm{id}\omega'}{2\pi}g_{\mathrm{m}}(\omega'-\omega,\mathbf{p}-\mathbf{q})g_{\mathrm{m}}(\omega',\mathbf{p}) = \sum_{\substack{\mathbf{p}\,\mathrm{in}\\ \mathbf{p}-\mathbf{q}\,\mathrm{out}}}\left(\frac{1}{\omega+\varepsilon^{(\mathrm{m})}_{\mathbf{p}}-\varepsilon^{(\mathrm{m})}_{\mathbf{p}-\mathbf{q}}+\mathrm{i}\eta} +\frac{1}{-\omega+\varepsilon^{(\mathrm{m})}_{\mathbf{p}}-\varepsilon^{(\mathrm{m})}_{\mathbf{p}-\mathbf{q}}+\mathrm{i}\eta}\right). \tag{3.13}$$

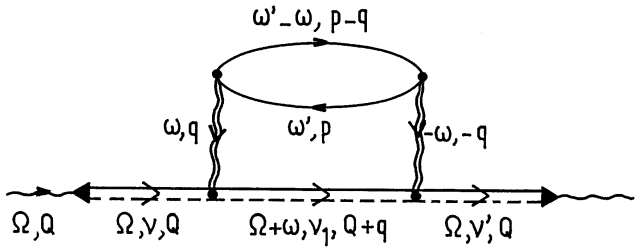

Fig. 5. Response function to second order in semiconductor-metal interaction, as given by exciton diagrams (Eq. (3.11)).

Inserting equation (3.13) into the last integral of equation (3.11), we get

$$\int\frac{\mathrm{id}\omega}{2\pi}G_{\nu_1}(\Omega+\omega,\mathbf{Q}+\mathbf{q})B_0(\omega,\mathbf{q}) = \sum_{\substack{\mathbf{p}\,\mathrm{in}\\ \mathbf{p}-\mathbf{q}\,\mathrm{out}}} G_{\nu_1}(\Omega+\varepsilon^{(\mathrm{m})}_{\mathbf{p}}-\varepsilon^{(\mathrm{m})}_{\mathbf{p}-\mathbf{q}},\mathbf{Q}+\mathbf{q}), \tag{3.14}$$

so that the exciton diagram procedure does give the same result as the one obtained from the direct calculation of the response function (Eq. (3.10)).

3.3 Electron and hole diagrams

In this last paragraph, we are going to recover the same result by using standard diagrams written with free electron and free hole propagators. Although basically "straightforward", this standard e-h diagram procedure is going to be quite heavy.

The first task is to draw all the possible diagrams, with as many V_{eh} interactions as we wish (including none), and two $(V_{\mathrm{em}}+V_{\mathrm{hm}})$ interactions, *i.e.* two V_{em}, or two V_{hm}, or one V_{em} and one V_{hm} (or the reverse), these electron-metal and hole-metal interactions having any possible position with respect to the V_{eh} interactions. All these possible diagrams are shown in Figure 6. In order to draw them up, we have been already forced to use the renormalized e-h Coulomb interaction and renormalized electron-photon interaction introduced in Section 2. In view of this Figure 6, it is clear that the obtention of the second order response function, as given in the compact form of equation (3.10), is going to be far from obvious.

One disturbing aspect of this problem is the fact that the processes in which the two semiconductor-metal interactions have not any e-h Coulomb interaction in between as in Figures 6a, b, c, seem to be topologically separated from the processes in which they have one or more V_{eh} interactions, as in Figures 6a′, b′, c′, c″. This somewhat disturbs our usual picture of an exciton seen as the result of zero, one, two... e-h Coulomb interactions, the zero term being part of these "ladder" processes. Some kind of algebraic "miracle" must take place somewhere in the calculation, in order to end up with a result, equation (3.10),

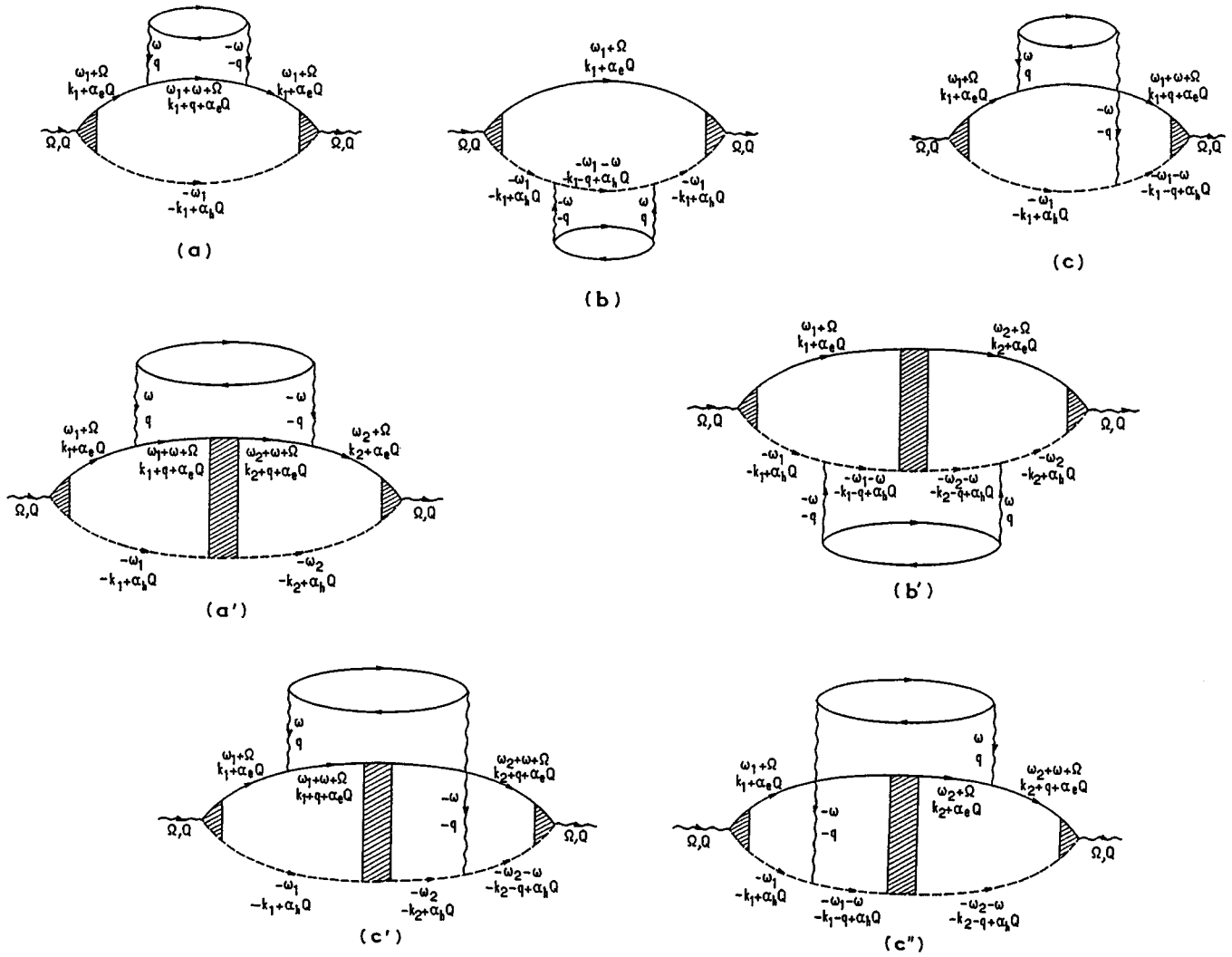

Fig. 6. Response function to second order in semiconductor-metal interaction, as given by standard e-h diagrams: (a) and (a′) with two electron-metal interactions (Eq. (3.16)); (b) and (b′) with two hole-metal interactions; (c), (c′) and (c″) with one electron-metal and one hole-metal interaction (Eqs. (3.22) and (3.24)).

in which everything is written in terms of (full) excitons, with the "zero" order term included along with the higher order ones.

3.3.1 Terms in V^2_{em} or V^2_{hm}

The terms which contain two electron-metal interactions correspond to the diagrams 6a and 6a′. They are somewhat similar except for the second $g_e(\omega_1+\Omega, \mathbf{k}_1+\alpha_e\mathbf{Q})$ propagator of Figure 6a which, in 6a′, is replaced by

$$\tilde{g}_e(\omega,\Omega,\mathbf{k}_1,\mathbf{k}_2,\mathbf{q},\mathbf{Q}) = W_{\mathbf{k}_1+\alpha_h\mathbf{q},\mathbf{k}_2+\alpha_h\mathbf{q}}(\omega+\Omega,\mathbf{q}+\mathbf{Q}) \times \int \frac{\mathrm{id}\omega_2}{2\pi} g_e(\omega_2+\Omega,\mathbf{k}_2+\alpha_e\mathbf{Q}) \times g_e(\omega_2+\omega+\Omega,\mathbf{k}_2+\mathbf{q}+\alpha_e\mathbf{Q}) g_h(-\omega_2,-\mathbf{k}_2+\alpha_h\mathbf{Q}). \quad (3.15)$$

The contribution of the two diagrams 6a and 6a′ can thus be written as

$$S_{a+a'}(\Omega,\mathbf{Q}) = \sum_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{q}} A^{(\mathrm{in})}_{\mathbf{k}_1}(\Omega,\mathbf{Q}) A^{(\mathrm{out})}_{\mathbf{k}_2}(\Omega,\mathbf{Q}) \tilde{V}^2_{\mathbf{q}} \times \int \frac{\mathrm{id}\omega}{2\pi} B_0(\omega,\mathbf{q}) I_e(\omega,\Omega,\mathbf{k}_1,\mathbf{k}_2,\mathbf{q},\mathbf{Q}), \quad (3.16)$$

$$I_e(\omega,\Omega,\mathbf{k}_1,\mathbf{k}_2,\mathbf{q},\mathbf{Q}) = \int \frac{\mathrm{id}\omega_1}{2\pi} g_e(\omega_1+\Omega,\mathbf{k}_1+\alpha_e\mathbf{Q}) \times g_h(-\omega_1,-\mathbf{k}_1+\alpha_h\mathbf{Q}) g_e(\omega_1+\omega+\Omega,\mathbf{k}_1+\mathbf{q}+\alpha_e\mathbf{Q}) \times [\delta_{\mathbf{k}_1\mathbf{k}_2} g_e(\omega_1+\Omega,\mathbf{k}_1+\alpha_e\mathbf{Q}) + \tilde{g}_e(\omega,\Omega,\mathbf{k}_1,\mathbf{k}_2,\mathbf{q},\mathbf{Q})]. \quad (3.17)$$

By using equation (2.9) for $g_{\rm e,h}$, we can perform the integration over ω_2 in equation (3.15), which gives

$$\tilde{g}_{\rm e}(\omega,\Omega,\mathbf{k}_1,\mathbf{k}_2,\mathbf{q},\mathbf{Q}) = W_{\mathbf{k}_1+\alpha_{\rm h}\mathbf{q},\mathbf{k}_2+\alpha_{\rm h}\mathbf{q}}(\omega+\Omega,\mathbf{q}+\mathbf{Q}) \times G_{\rm eh}(\Omega,\mathbf{k}_2,\mathbf{Q})G_{\rm eh}(\omega+\Omega,\mathbf{k}_2+\alpha_{\rm h}\mathbf{q},\mathbf{q}+\mathbf{Q}), \quad (3.18)$$

where $G_{\rm eh}$ is the free e-h pair propagator defined in equation (2.10). Inserting this result in equation (3.17), and performing the integration over ω_1 in the same way, we get

$$I_{\rm e}(\omega,\Omega,\mathbf{k}_1,\mathbf{k}_2,\mathbf{q},\mathbf{Q}) = G_{\rm eh}(\Omega,\mathbf{k}_1,\mathbf{q})G_{\rm eh}(\Omega,\mathbf{k}_2,\mathbf{q}) \times G_{\rm eh}(\omega+\Omega,\mathbf{k}_1+\alpha_{\rm h}\mathbf{q},\mathbf{q}+\mathbf{Q}) \times [\delta_{\mathbf{k}_1\mathbf{k}_2} + W_{\mathbf{k}_1+\alpha_{\rm h}\mathbf{q},\mathbf{k}_2+\alpha_{\rm h}\mathbf{q}}(\omega+\Omega,\mathbf{q}+\mathbf{Q}) \times G_{\rm eh}(\omega+\Omega,\mathbf{k}_2+\alpha_{\rm h}\mathbf{q},\mathbf{q}+\mathbf{Q})]. \quad (3.19)$$

Then the (first) necessary "miracle" takes place: A look at equation (2.18) shows that the renormalized e-h interaction $W_{\mathbf{k}_1+\alpha_{\rm h}\mathbf{q},\mathbf{k}_2+\alpha_{\rm h}\mathbf{q}}$ contains a $\delta_{\mathbf{k}_1\mathbf{k}_2}$ term which cancels the $\delta_{\mathbf{k}_1\mathbf{k}_2}$ term of equation (3.19) (originating from diagram 6a). By inserting equation (2.18) into equation (3.19), we are left with $I_{\rm e}$ written in terms of G_ν and $G_{\rm eh}$ while we expect $S_{a+a'}$ to depend on G_ν only. We then note that $A^{(\rm in)}_{\mathbf{k}_1}$ and $A^{(\rm out)}_{\mathbf{k}_2}$ appearing in equation (3.16) also contain $G_{\rm eh}$ factors: By using equations (2.20) and (2.21), we find that all the $G_{\rm eh}$ factors finally disappear, and we get

$$A^{(\rm in)}_{\mathbf{k}_1}(\Omega,\mathbf{Q})A^{(\rm out)}_{\mathbf{k}_2}(\Omega,\mathbf{Q})I_{\rm e}(\omega,\Omega,\mathbf{k}_1,\mathbf{k}_2,\mathbf{q},\mathbf{Q}) = \sum_{\nu,\nu',\nu_1} A^*_{\nu'}A_\nu\phi^*_{\nu'}(\mathbf{k}_2)\phi_\nu(\mathbf{k}_1)\phi^*_{\nu_1}(\mathbf{k}_1+\alpha_{\rm h}\mathbf{q})\phi_{\nu_1}(\mathbf{k}_2+\alpha_{\rm h}\mathbf{q}) \times G_{\nu'}(\Omega,\mathbf{Q})G_\nu(\Omega,\mathbf{Q})G_{\nu_1}(\omega+\Omega,\mathbf{q}+\mathbf{Q}), \quad (3.20)$$

which depends on G_ν only. Inserting this result into equation (3.16) and using equations (1.33) and (3.14), we finally get

$$S_{a+a'}(\Omega,\mathbf{Q}) = \sum_{\nu,\nu',\nu_1}\sum_{\mathbf{q}} A^*_{\nu'}A_\nu\tilde{V}^2_{\mathbf{q}}\langle x_{\nu'}|{\rm e}^{-{\rm i}\alpha_{\rm h}\mathbf{q}.\mathbf{r}}|x_{\nu_1}\rangle \times \langle x_{\nu_1}|{\rm e}^{{\rm i}\alpha_{\rm h}\mathbf{q}.\mathbf{r}}|x_\nu\rangle G_{\nu'}(\Omega,\mathbf{Q})G_\nu(\Omega,\mathbf{Q}) \times \sum_{\substack{\mathbf{p}\,\rm in\\ \mathbf{p}-\mathbf{q}\,\rm out}} G_{\nu_1}(\Omega+\varepsilon^{(\rm m)}_{\mathbf{p}}-\varepsilon^{(\rm m)}_{\mathbf{p}-\mathbf{q}},\mathbf{Q}+\mathbf{q}). \quad (3.21)$$

We already see that $S_{a+a'}(\Omega,\mathbf{Q})$ does look like $S^{(2)}(\Omega,\mathbf{Q})$, except for $\gamma_{\nu_1\nu}(\mathbf{q})$ which is here replaced by $\langle x_{\nu_1}|{\rm e}^{{\rm i}\alpha_{\rm h}\mathbf{q}.\mathbf{r}}|x_\nu\rangle$, *i.e.* a part of $\gamma_{\nu_1\nu}(\mathbf{q})$ only, as can be seen from equation (1.36).

The terms with two $V_{\rm hm}$ correspond to the diagrams 6b and 6b′. A similar calculation shows that their contribution, $S_{b+b'}(\Omega,\mathbf{Q})$, is obtained from $S_{a+a'}(\Omega,\mathbf{Q})$ by replacing $\alpha_{\rm h}$ by $(-\alpha_{\rm e})$.

3.3.2 Terms in $V_{\rm em}V_{\rm hm}$

They correspond to diagram 6c, and to diagrams 6c′ and 6c″ which are different, since the processes in which $V_{\rm em}$ acts before $V_{\rm hm}$ are not equivalent to the processes in which it acts after $V_{\rm hm}$. We could think that, as for the terms in $V^2_{\rm em}$, it would be convenient to consider diagrams 6c and 6c′ together in order to have the same "miraculous" simplification appearing in a natural way. However we could as well consider diagrams 6c and 6c″ together. The trouble is that diagram 6c appears once only. In fact, if we look at diagram 6c more carefully, we see that it contains two $g_{\rm e}$ and two $g_{\rm h}$, while diagram 6a contained three $g_{\rm e}$ and one $g_{\rm h}$. Integration over ω_1 thus leads to one term only in the contribution of diagram 6a, while it leads to two terms in the contribution of diagram 6c. It turns out that one of these terms makes one of the "miraculous" simplifications with the $\delta_{\mathbf{k}_1\mathbf{k}_2}$ term of diagram 6c′, while the other term cancels the $\delta_{\mathbf{k}_1\mathbf{k}_2}$ term of diagram 6c″.

More precisely, the contribution of diagram 6c reads

$$S_c(\Omega,\mathbf{Q}) = -\sum_{\mathbf{k}_1,\mathbf{q}} A^{(\rm in)}_{\mathbf{k}_1}(\Omega,\mathbf{Q})A^{(\rm out)}_{\mathbf{k}_1+\mathbf{q}}(\Omega,\mathbf{Q})\tilde{V}^2_{\mathbf{q}} \times \int\frac{{\rm d}\omega_1}{2\pi}\int\frac{{\rm d}\omega}{2\pi}B_0(\omega,\mathbf{q})g_{\rm e}(\omega_1+\Omega,\mathbf{k}_1+\alpha_{\rm e}\mathbf{Q}) \times g_{\rm e}(\omega_1+\omega+\Omega,\mathbf{k}_1+\mathbf{q}+\alpha_{\rm e}\mathbf{Q})g_{\rm h}(-\omega_1,-\mathbf{k}_1+\alpha_{\rm h}\mathbf{Q}) \times g_{\rm h}(-\omega_1-\omega,-\mathbf{k}_1-\mathbf{q}+\alpha_{\rm h}\mathbf{Q}). \quad (3.22)$$

After integration over ω_1 and ω, we get it as the sum of two terms:

$$S_c(\Omega,\mathbf{Q}) = -\sum_{\mathbf{k}_1,\mathbf{q}} A^{(\rm in)}_{\mathbf{k}_1}(\Omega,\mathbf{Q})A^{(\rm out)}_{\mathbf{k}_1+\mathbf{q}}(\Omega,\mathbf{Q}) \times \tilde{V}^2_{\mathbf{q}}G_{\rm eh}(\Omega,\mathbf{k}_1,\mathbf{Q})G_{\rm eh}(\Omega,\mathbf{k}_1+\mathbf{q},\mathbf{Q}) \times \sum_{\substack{\mathbf{p}\,\rm in\\ \mathbf{p}-\mathbf{q}\,\rm out}} [G_{\rm eh}(\Omega+\varepsilon^{(\rm m)}_{\mathbf{p}}-\varepsilon^{(\rm m)}_{\mathbf{p}-\mathbf{q}},\mathbf{k}_1+\alpha_{\rm h}\mathbf{q},\mathbf{Q}+\mathbf{q}) + G_{\rm eh}(\Omega+\varepsilon^{(\rm m)}_{\mathbf{p}}-\varepsilon^{(\rm m)}_{\mathbf{p}-\mathbf{q}},\mathbf{k}_1+\alpha_{\rm e}\mathbf{q},\mathbf{Q}-\mathbf{q})]. \quad (3.23)$$

If we now turn to diagram 6c′, its contribution reads

$$S_{c'}(\Omega,\mathbf{Q}) = -\sum_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{q}} A^{(\rm in)}_{\mathbf{k}_1}(\Omega,\mathbf{Q})A^{(\rm out)}_{\mathbf{k}_2+\mathbf{q}}(\Omega,\mathbf{Q}) \times \tilde{V}^2_{\mathbf{q}}\int\frac{{\rm d}\omega_1}{2\pi}\int\frac{{\rm d}\omega_2}{2\pi}\int\frac{{\rm d}\omega}{2\pi}B_0(\omega,\mathbf{q}) \times W_{\mathbf{k}_1+\alpha_{\rm h}\mathbf{q},\mathbf{k}_2+\alpha_{\rm h}\mathbf{q}}(\omega+\Omega,\mathbf{q}+\mathbf{Q})g_{\rm e}(\omega_1+\Omega,\mathbf{k}_1+\alpha_{\rm e}\mathbf{Q}) \times g_{\rm e}(\omega_1+\omega+\Omega,\mathbf{k}_1+\mathbf{q}+\alpha_{\rm e}\mathbf{Q})g_{\rm e}(\omega_2+\omega+\Omega,\mathbf{k}_2+\mathbf{q}+\alpha_{\rm e}\mathbf{Q}) \times g_{\rm h}(-\omega_1,-\mathbf{k}_1+\alpha_{\rm h}\mathbf{Q})g_{\rm h}(-\omega_2,-\mathbf{k}_2+\alpha_{\rm h}\mathbf{Q}) \times g_{\rm h}(-\omega_2-\omega,-\mathbf{k}_2-\mathbf{q}+\alpha_{\rm h}\mathbf{Q}). \quad (3.24)$$

We integrate over ω_1 and ω_2 and replace W by its explicit value given in equation (2.18). This leads to

$$S_{c'}(\Omega,\mathbf{Q}) = -\sum_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{q}} A^{(\mathrm{in})}_{\mathbf{k}_1}(\Omega,\mathbf{Q}) A^{(\mathrm{out})}_{\mathbf{k}_2+\mathbf{q}}(\Omega,\mathbf{Q})\tilde{V}^2_{\mathbf{q}} \times \int \frac{\mathrm{i}d\omega}{2\pi} B_0(\omega,\mathbf{q}) G_{\mathrm{eh}}(\Omega,\mathbf{k}_1,\mathbf{Q}) G_{\mathrm{eh}}(\Omega,\mathbf{k}_2+\mathbf{q},\mathbf{Q}) \times \Bigg[-\delta_{\mathbf{k}_1\mathbf{k}_2} G_{\mathrm{eh}}(\omega+\Omega,\mathbf{k}_1+\alpha_{\mathrm{h}}\mathbf{q},\mathbf{q}+\mathbf{Q}) + \sum_{\nu_1} G_{\nu_1}(\omega+\Omega,\mathbf{q}+\mathbf{Q})\phi^*_{\nu_1}(\mathbf{k}_1+\alpha_{\mathrm{h}}\mathbf{q})\phi_{\nu_1}(\mathbf{k}_2+\alpha_{\mathrm{h}}\mathbf{q}) \Bigg]. \tag{3.25}$$

We first consider the term in $\delta_{\mathbf{k}_1\mathbf{k}_2}$ and make the integration over ω. As claimed above, it gives a contribution, $S_{c'1}(\Omega,\mathbf{Q})$, which cancels exactly the first term of $S_c(\Omega,\mathbf{Q})$ given in equation (3.23). As to the remaining term of $S_{c'}$, after integration over ω, it reads

$$S_{c'2}(\Omega,\mathbf{Q}) = -\sum_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{q}} A^{(\mathrm{in})}_{\mathbf{k}_1}(\Omega,\mathbf{Q}) A^{(\mathrm{out})}_{\mathbf{k}_2+\mathbf{q}}(\Omega,\mathbf{Q}) \times \tilde{V}^2_{\mathbf{q}} G_{\mathrm{eh}}(\Omega,\mathbf{k}_1,\mathbf{Q}) G_{\mathrm{eh}}(\Omega,\mathbf{k}_2+\mathbf{q},\mathbf{Q}) \times \sum_{\nu_1} \phi^*_{\nu_1}(\mathbf{k}_1+\alpha_{\mathrm{h}}\mathbf{q})\phi_{\nu_1}(\mathbf{k}_2+\alpha_{\mathrm{h}}\mathbf{q}) \times \sum_{\substack{\mathbf{p}\,\mathrm{in}\\ \mathbf{p}-\mathbf{q}\,\mathrm{out}}} G_{\nu_1}(\Omega+\varepsilon^{(\mathrm{m})}_{\mathbf{p}}-\varepsilon^{(\mathrm{m})}_{\mathbf{p}-\mathbf{q}},\mathbf{Q}+\mathbf{q}). \tag{3.26}$$

We then replace $A^{(\mathrm{in})}_{\mathbf{k}_1}$ and $A^{(\mathrm{out})}_{\mathbf{k}_2+\mathbf{q}}$ by their explicit values given in equations (2.20) and (2.21), and perform the summations over $\mathbf{k}_1$ and $\mathbf{k}_2$ using equation (1.33). We finally get

$$S_{c'2}(\Omega,\mathbf{Q}) = -\sum_{\nu,\nu',\nu_1}\sum_{\mathbf{q}} A^*_{\nu'}A_{\nu}\tilde{V}^2_{\mathbf{q}} \times \langle x_{\nu'}|\mathrm{e}^{\mathrm{i}\alpha_{\mathrm{e}}\mathbf{q}\cdot\mathbf{r}}|x_{\nu_1}\rangle\langle x_{\nu_1}|\mathrm{e}^{\mathrm{i}\alpha_{\mathrm{h}}\mathbf{q}\cdot\mathbf{r}}|x_{\nu}\rangle G_{\nu}(\Omega,\mathbf{Q}) G_{\nu'}(\Omega,\mathbf{Q}) \times \sum_{\substack{\mathbf{p}\,\mathrm{in}\\ \mathbf{p}-\mathbf{q}\,\mathrm{out}}} G_{\nu_1}(\Omega+\varepsilon^{(\mathrm{m})}_{\mathbf{p}}-\varepsilon^{(\mathrm{m})}_{\mathbf{p}-\mathbf{q}},\mathbf{Q}+\mathbf{q}). \tag{3.27}$$

A similar calculation done for the contribution of diagram 6c″ shows that the first term, $S_{c''1}(\Omega,\mathbf{Q})$, cancels the second term of $S_c(\Omega,\mathbf{Q})$ as given in equation (3.23), while the second term, $S_{c''2}(\Omega,\mathbf{Q})$ is obtained from $S_{c'2}(\Omega,\mathbf{Q})$ by changing α_{e} into $(-\alpha_{\mathrm{h}})$ and α_{h} into $(-\alpha_{\mathrm{e}})$.

3.3.3 Terms in $(V_{em} + V_{hm})^2$

Their contributions correspond to diagrams 6a to 6c″. Using equation (1.36), it is easy to check that their sum,

$$S^{(2)}(\Omega,\mathbf{Q}) = S_{a+a'}(\Omega,\mathbf{Q}) + S_{b+b'}(\Omega,\mathbf{Q}) + S_{c'2}(\Omega,\mathbf{Q}) + S_{c''2}(\Omega,\mathbf{Q}), \tag{3.28}$$

gives the same expression as the one obtained in equation (3.10): After a lot of efforts, we have proved that the method using electron and hole diagrams does give the same result as the one using exciton diagrams. Very honestly, we would have hardly found all these "miraculous" simplifications if we had not been convinced by the exciton diagrams that they must exist.

4 Conclusion

We have introduced an exciton diagram procedure and have shown on two specific examples (a semiconductor without any interaction, and a semiconductor interacting with a distant metal to second order in this interaction) that it allows to recover in an extremely elegant way the results obtained from standard diagrams with free electrons and holes.

This procedure is based on the following rules:

- We associate to each exciton line an exciton propagator given by

$$G_{\nu}(\omega,\mathbf{q}) = \frac{1}{\omega-\varepsilon_{\nu}-E_{\mathbf{q}}+\mathrm{i}\eta}\cdot \tag{4.1}$$

  Note that this exciton propagator only depends on the $(\nu,\mathbf{q})$ quantum numbers characterizing the exciton state, by contrast with the two-particle Green's function, unproperly called exciton propagator, which depends on the "in" and "out" electron and hole momenta.
- We associate to the exciton-photon interaction, the vertex

$$A_{\nu} = A\sqrt{\mathcal{V}}\langle x_{\nu}|\mathbf{r}=\mathbf{0}\rangle\cdot \tag{4.2}$$

- In the case of a semiconductor-metal interaction, which appears through interactions with free electrons and free holes, we determine the corresponding vertex by calculating

$$V|X_{\nu,\mathbf{q}}\rangle\otimes|M\rangle\cdot \tag{4.3}$$

  In order to do it, we "open" the exciton into e-h pairs through

$$|X_{\nu,\mathbf{q}}\rangle = B^+_{\nu,\mathbf{q}}|0\rangle = \sum_{\mathbf{k}}\phi_{\nu}(\mathbf{k})a^+_{\mathbf{k}+\alpha_{\mathrm{e}}\mathbf{q}}b^+_{-\mathbf{k}+\alpha_{\mathrm{h}}\mathbf{q}}|v\rangle\cdot \tag{4.4}$$

  We calculate the action of V on these free e-h pairs, and we write them back in terms of excitons by using

$$a^+_{\mathbf{k}_{\mathrm{e}}}b^+_{\mathbf{k}_{\mathrm{h}}} = \sum_{\nu}\phi^*_{\nu}(\alpha_{\mathrm{h}}\mathbf{k}_{\mathrm{e}}-\alpha_{\mathrm{e}}\mathbf{k}_{\mathrm{h}})B^+_{\nu,\mathbf{k}_{\mathrm{e}}+\mathbf{k}_{\mathrm{h}}}. \tag{4.5}$$

  This leads to

$$V|X_{\nu,\mathbf{q}}\rangle\otimes|M\rangle = \sum_{\nu',\mathbf{q}'}\tilde{V}_{\mathbf{q}'}\gamma_{\nu'\nu}(\mathbf{q}')|X_{\nu',\mathbf{q}+\mathbf{q}'}\rangle\otimes|M'(\mathbf{q}')\rangle, \tag{4.6}$$

  from which we extract the value of the exciton-metal vertex interaction as being $\tilde{V}_{\mathbf{q}'}\gamma_{\nu'\nu}(\mathbf{q}')$.

- At each vertex interaction, we force $(\omega, \mathbf{q})$ conservation, and we sum up over all the remaining free variables.

We have here proved, by calculating explicitly the second order term in the semiconductor-metal interaction, that this exciton diagram procedure gives the same result as the one obtained from a very safe but extremely tedious calculation using e-h diagrams. If we want to go beyond second order in the semiconductor-metal interaction, and classify the set of "dominant" processes in these interactions with a distant metal, as done in an independent publication [11], it is clearly completely hopeless to use this e-h diagram procedure. On that respect, the exciton diagrams provide a clear and enlightening picture of what physically happens. Let us mention that the procedure "without diagram" we also give here, is also quite inappropriate to an extension to high order processes, as it is basically an algebraic version of the e-h diagram (Green's function) procedure.

By considering the interaction of an exciton with distant carriers, we have avoided the extremely tricky consequences of Pauli exclusion between two identical electrons (or holes). The derivation of the correct vertex interaction which has to be used in exciton diagrams for problems dealing with two or more identical carriers from which an exciton can be made, will be the subject of forthcoming publications.

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15. For simplicity we assume that all the dielectric constants are equal to ε. This is reasonable for quantum well structures, and avoids extensive complications due to dielectric effects.